revised version

Lab 4:

DIFFRACTION GRATINGS AND PRISMS (3 Lab Periods)

Objectives Calibrate a diffraction grating using a spectral line of known wavelength. With the calibrated grating, determine the wavelengths of several other spectral lines. Determine the chromatic resolving power of the grating. Determine the dispersion curve (refractive index as a function of wavelength) of a glass prism.

References Hecht, sections 3.5, 5.5, 10.2.8; tables 3.3 and 6.2

(A) Basic Equations

We will discuss diffraction gratings in greater detail later in the course. In this laboratory, you will need to use only two basic grating equations, and you will not need the details of the later discussion. The first equation should be familiar to you from an introductory Physics course and describes the angular positions of the principal maxima of order m for light of wavelength λ.

$$a \sin\theta_m = m\lambda \qquad (4.1)$$

where a is the separation between adjacent grooves in the grating.

The other, which may not be as familiar, is the equation for the chromatic resolving power R_m in the diffraction order m when N grooves in the grating are illuminated.

$$R_m = \left|\lambda/(\Delta\lambda)_{\min}\right| = mN \qquad (4.2)$$

where $(\Delta\lambda)_{\min}$ is the smallest wavelength difference for which two spectral lines, one of wavelength λ and the other of wavelength $\lambda + \Delta\lambda$, will just be resolved. The absolute value insures that R will be a positive quantity for either sign of $\Delta\lambda$. If $\Delta\lambda$ is small, as it will be in this experiment, it does not matter whether you use λ, $\lambda + \Delta\lambda$, or the average value in the numerator. For example, the D lines in sodium vapor are a yellow doublet (λ_{D1} = 589.593 nm and λ_{D2} = 588.996 nm), whose two components are separated by 0.597 nm. The average wavelength is 589.295 nm. Thus the minimum resolving power required to distinguish the two components is given by R = 589.295/0.597 = 987.

(B) **Equipment:** Divided Circle Spectrometer, diffraction grating, variable-width slit, Hydrogen lamp, Sodium Lamp, Helium lamp, glass prism.

(C) **LAB SAFETY: Never look through the telescope directly at any line source.**

(D) Initial Considerations

The resolution actually obtained in an instrument, such as a prism or grating spectrometer, may be limited by many things: the slit width, the quality of the lenses, *etc*. Even with an infinitely narrow slit and perfect optics, diffraction by the finite aperture of the dispersive element imposes a fundamental limitation on the resolving power of the instrument. For a grating oriented perpendicular to the beam that is illuminated over its full width, the effective aperture is just the width of the patterned region of the grating. However, if the collimator or telescope have lenses of diameter less than the effective aperture of either prism or grating, they will determine the limiting aperture. In this experiment you will control the size of the effective aperture by deliberately inserting a slit of variable width between the collimator and the grating, thus limiting the width of the illuminated region.

The theoretical resolving power of a transmission grating is given by $R = mN$, Eq. (4.2), where m is the order of the principal maximum and N is the total number of grooves in the grating that are actually illuminated by the source. For example, if a grating has 5,000 lines/cm and you illuminate a 1 cm width, then N = 5,000 and the theoretical resolving power $R = mN$ in 2nd order (m = 2) would be 2×5,000 = 10,000. If you illuminated only a 2 mm width of this grating, the number of illuminated lines (1,000) would be just about enough to resolve the sodium D lines in first order, and would be more than adequate in second order (mN = 2,000). Note that the resolving power of a grating is independent of wavelength.

This experiment makes use of the divided-circle spectrometer. Others may have used your spectrometer in between lab periods, so you should begin each laboratory period by making the initial adjustments to the collimator and telescope as described below. *Be sure you use the same spectrometer, diffraction grating, and prism throughout this experiment.* The gratings, prisms, and spectrometers are labeled. Be sure to record the labels of each. The gratings and prisms may not be identical; if you make some of your measurements with one and some with another you may introduce systematic errors into your results.

(E) Initial Adjustments (see Fig. 4.1)

a. Focusing the Telescope

Direct the telescope at a bright field (*e.g.*, a white image board illuminated by an incandescent lamp) and focus the eyepiece on the cross-hairs.

Take the spectrometer to a window and look through the telescope at a distant object (or to the end of a long corridor and look at an object at the opposite end of the corridor). With the rack and pinion adjustment, move the telescope in and out until the object is in good focus. If these two adjustments are correct there should be no relative motion of the cross-hairs and the image of the distant object as you move your eye back-and-forth across the eyepiece. The telescope now focuses parallel light at the cross-hair plane.

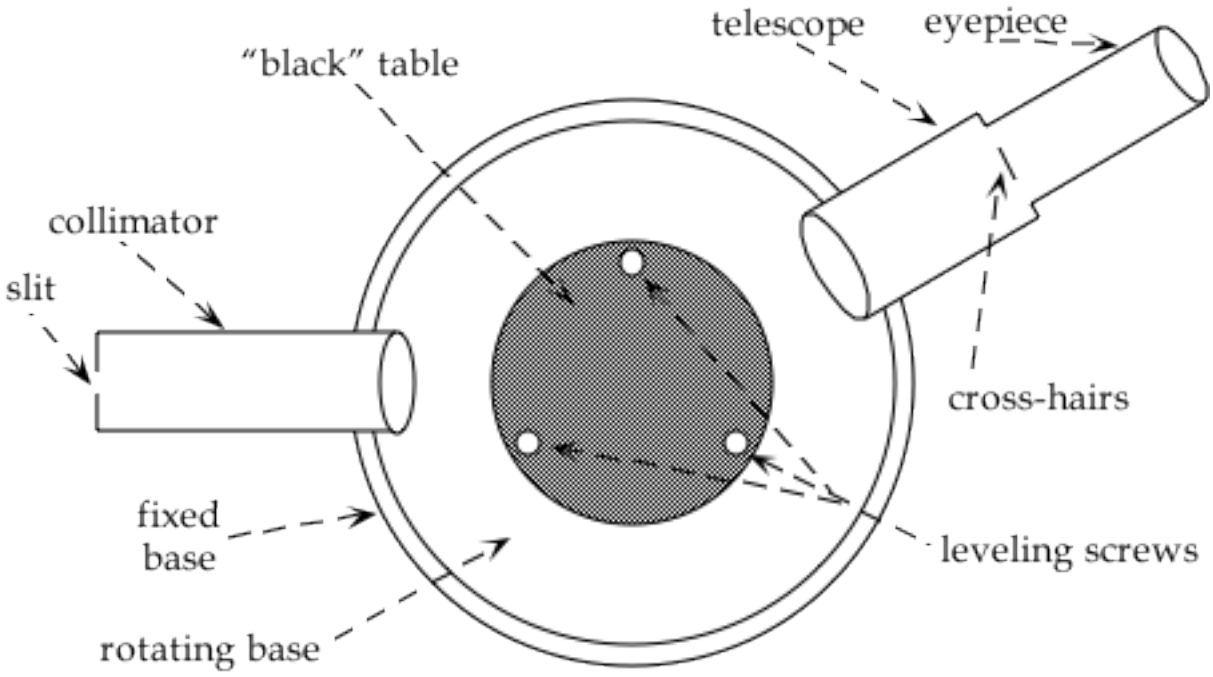

Figure 4.1: Divided-circle spectrometer

b. Focusing the Collimator

Illuminate the slit with an *incandescent* light source. Rotate the telescope (adjusted as described above) so as to look into the collimator. Adjust the collimator until the slit is in focus. Light diverging from the slit will now emerge from the collimator as a parallel beam.

c. Leveling the Black Table

The plane of the black table should be perpendicular to the axis of rotation. Using a bubble level, make sure that the spectrometer base is level. Then adjust the black table until it is also level.

d. Aligning the Diffraction Grating

For the portions of this laboratory which utilize the grating, fix the grating on the table, directly over the center of rotation, with the good (ruled) side of the grating facing away from the collimator (i.e. the unruled, plane glass side of the grating facing toward the collimator). For accurate measurements, it is essential to adjust the spectrometer so that the grating is perpendicular to the beam from the collimator. A recommended procedure is as follows:

1. Align the telescope with the collimator. Make the slit as narrow as possible, and center the cross-hairs on the image of the slit. Lock the telescope in place by tightening the small black knob, located more or less directly below the telescope mount. This defines the 0° direction.

2. Unlock the angle scale, by loosening the small black knob on the left side of the spectrometer base, and rotate the scale until it reads exactly 45°. Lock the angle scale at 45°.

3. Keeping the angle scale locked, unlock the telescope and rotate it around to the right, until the angle reading is exactly 135°. Lock the telescope at 135°.

4. Keeping both the angle scale and the telescope locked in their current positions, unlock the black table, using the long knob between the black table and the angle scale.

5. Looking through the telescope, rotate the black table until the good (ruled) side of the grating reflects an image of the slit into the telescope. When the image of the slit is centered on the cross hairs, lock the black table in place.

6. Unlock the telescope and swing it back to exactly 45°. Lock the telescope at 45°.

7. Unlock the angle scale and rotate the angle scale back to 0°. Lock the angle scale at 0°.

8. Look through the eyepiece and check to be sure the cross hairs are centered in the image of the slit. (Fine adjustment of less than 1 minute of arc should be all that is needed.)

(F) Procedure

a. Calibration of a Diffraction Grating

Using the red line in the Hydrogen spectrum (Balmer-α) as a reference of known wavelength (λ = 656.28 nm), measure the angles of the first and second-order principal maxima on each side. If the diffraction grating is properly aligned, the angles for a given wavelength, in a given order of interference, should be the same on both sides, to within a few minutes or arc. Using these measurements, determine the grating spacing and the number of lines per mm. You should obtain a result that is close to the nominal value of 600 lines per mm.

b. Wavelength Measurements with a Grating Spectrometer

Measure the angles of the first and second-order principal maxima of the prominent lines in the Helium spectrum. Then determine the wavelengths of these lines. You will use these spectral lines for measurements on a dispersing prism in section **d**. Check your results for λ against those in the wavelength tables in the laboratory, or in Table 3.3 in Hecht (p. 70).

c. Chromatic Resolving Power of a Diffraction Grating

Diffraction of light that passes an aperture may limit the resolution of an instrument. However, the resolution of interest in that situation is the *spatial* resolution, *i.e.*, the ability of the instrument to resolve two spatially separated objects that are emitting light of the same wavelength. In

distinction, the resolution of interest in this experiment is the *chromatic* resolution, *i.e.*, the ability of the instrument to resolve two different wavelengths emitted by atoms in the same source (no spatial separation at all).

In this experiment, you introduce a slit between the collimator and the grating that allows you to vary the width of the beam that illuminates the grating. The number of illuminated lines is then determined by an aperture that you can control and measure, rather than by elements such as the collimator whose apertures are either fixed or difficult to measure. The beam through the slit may be treated in the limit of geometrical optics; diffraction by the slit is not important.

The grating should be perpendicular to the beam from the collimator. Using the Na D lines as the source, place a variable slit between the collimator and grating and adjust the slit until the two lines of the doublet are just barely resolved. Then measure the slit width. As you narrow the slit, you reduce the number of illuminated lines of the grating until the Na D lines are just resolved. Because you know both λ and $\Delta\lambda$, you can compute the resolving power $|\lambda/\Delta\lambda|$ for this situation. You can determine the total number of illuminated lines N from the slit width and the number of lines per unit length of the grating, which you measured earlier.

Make several measurements, with each partner adjusting the slit an equal number of times. If you use two adjustable slits, one partner can be setting one slit in the spectrometer while the other is measuring the width of the other slit. Treat each partner's measurements separately to check for systematic differences.

Compare your value of $R = mN$ with the value of $|\lambda/\Delta\lambda|$ calculated for the Na doublet. Do this in both first and second order. Can you observe the Na D lines in third order? If not, explain why.

Technical hint: For the measurement of R, you should have the system carefully adjusted, and you should use the narrowest possible collimator slit width.

d. Dispersion of a Glass Prism

(1) Measuring the Prism Angle, α

Set up a prism on the black table of the spectrometer, as follows

If your spectroscope is still set up with the diffraction grating, carefully remove the grating and grating holder from the black table, and store them safely in the spectrometer accessories box. Attach the prism clamp to the black table, using the thumb screws and the appropriate pair of threaded holes in the black table. Position the prism on the black table such that the base of the prism is snug up against the prism clamp, and the apex of the prism points away from the clamp, toward the opposite side of the table.

Measure the prism angle, α, using the following procedure.

Unlock the black table and rotate it such that the apex of the prism points directly toward the collimator (i.e. the base of the prism should be approximately perpendicular to the collimator axis). Illuminate the collimator slit with a Sodium lamp. Lock the black table in this position. If the prism has been positioned properly, light from the illuminated slit should be reflected more or less symmetrically from both sides of the prism. With your *unaided* eye, look for the image of the slit reflected from each side of the prism. (If you can't see the reflected image of the slit with your unaided eye, you will have little hope of being able to find it with the telescope.) When you have found the reflected image of the slit by eye, use the telescope to make careful measurements of the angle at which the image appears. (For each measurement, read the vernier scale to the nearest minute of arc.)

Take several repeated measurements of the angle of the reflected image on both side of the prism. Determine the average and the uncertainty for these two sets of measurements. The apex angle, α, is half the difference between the angles of the two reflected images. Why is the uncertainty in reading the vernier scale not an adequate estimate of the uncertainty of α?

(2) Angle of Minimum Deviation for the Na-D lines

Hint: **Carrying out the following calculations *before* you come to the lab will save you time.**

An expression for the angle of minimum deviation δ_m for a prism can be derived in terms of the apex angle and the index of refraction. (See Hecht, Section 5.5.1, and solve equation 5.54 for δ_m.) Using this expression, determine δ_m for a prism whose apex angle is the same as the angle you measured for your prism, in section (1) above. Do this calculation for a range (about 6) of the glasses described in Hecht, Table 6.2, p. 270. These refer to the average wavelength of the Na-D lines. Then measure δ_m for this wavelength (λ = 589.3 nm). This result should be quite close to one of the values you have calculated for the various different glasses. If it is, you can be reasonably certain that you measured δ_m correctly. This should also suggest the type of glass for your prism, and hence you can compute the range of δ_m that you would expect for measurements at different wavelengths.

(3) Dispersion Curve

Illuminate the spectrometer slit with a Helium lamp. Using the He lines, whose wavelengths you determined in section **b.**, measure δ_m for each line and find the corresponding indices of refraction from the relation

$$n = \frac{\sin\left(\frac{\alpha + \delta_m}{2}\right)}{\sin\left(\alpha/2\right)} \qquad (4.3)$$

The *dispersion curve* is a graph of n vs. λ. An empirical formula proposed by Cauchy can be used to fit the dispersion curve. This formula, a power series expansion of n in the variable $1/\lambda^2$, is described on p. 85 of Hecht. Retaining only the first two terms of the expansion, we have

$$n = n_0 + K\lambda^2 \tag{4.4}$$

where n_0 and K are constants for a particular glass.

Plot a graph of n vs. λ. Then plot another graph of n vs. $1/\lambda^2$ and, by fitting a straight line to the data, find n_0 and K. What are the dimensions of the constant K? Be sure to express it in appropriate units.

For each wavelength, take several measurements of δ_m and determine the average and the uncertainty of the set. Combine the uncertainties in the α and δ_m to determine the uncertainty in n.

(3) Phase Velocity and Group Velocity

In a non-dispersive medium, the phase velocity does not depend on wavelength and therefore, phase and group velocities are equal. In a dispersive medium, however, the phase velocity does depend on wavelength and can be written in terms of the wavelength-dependent index of refraction as $v_p = c/n(\lambda)$. In this case, the phase and group velocities will not be equal, and are related by the expression

$$v_g = v_p\left(1 + \frac{\lambda}{n}\frac{\mathrm{d}n}{\mathrm{d}\lambda}\right) \tag{4.5}$$

We can write the difference as

$$v_g - v_p = v_p\frac{\lambda}{n}\frac{\mathrm{d}n}{\mathrm{d}\lambda} = \frac{c\lambda}{n^2}\frac{\mathrm{d}n}{\mathrm{d}\lambda} \tag{4.6}$$

Determine an expression for $\mathrm{d}n/\mathrm{d}\lambda$ from the Cauchy relation, Eq. (4.4), and calculate the difference $v_g - v_p$ for several values of λ in the range of your measurements. Present your results in a graph.